External Network Modeling for Optimal Power Flow Applications

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Abstract --- Traditional external network modeling techniques have been widely used in the power industry to produce equivalent models that are suitable for power flow analysis and security assessment applications. These techniques cannot be used to produce models for Optimal Power Flow applications. In this paper, we present a new methodology that is suitable for security analysis and security control. The proposed method has been tested extensively under a wide variety of operating conditions using data from a large scale system. The results strongly indicate that the proposed method outperforms the Extended Ward method in producing equivalent models for security control and does not create an additional burden in maintaining and updating the EMS database.

Keywords: External Network Modeling, Security Analysis, Optimal Power Flow, Energy Management Systems

I. INTRODUCTION

The on-line security analysis functions in a modern Energy Management System (EMS) periodically process the steady-state results of hundred of contingencies to determine the security of the power system. These functions have served the industry well. However, recent changes in the utility business environment threaten to erode the traditionally high level of security by exposing the system to smaller security margins and resulting in transmission of electric energy over long distances in patterns other than those for which the transmission networks have been originally designed. These changes include the emergence of competitive energy marketplace, regulatory reforms, increased pressure for essentially uncontrolled access to the bulk power transmission networks, rising cost of money and stricter environment regulations for generation and transmission construction. The current situation means that there is a need, more than ever, to maintain the reliability of electric energy supply and the high level of system security.

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For accurate results the analysis must be done on a representation of the total interconnected system. The reason is that power systems are interconnected and an external network model that accounts for the response of the “external" system must be added to the ‘internal’ system. The internal system is the monitored part of the power system and usually consists of one’s own system and a portion of other systems which is “electrically closed” to that system. The rest of the interconnected system is called external system, and is usually unmetered and “electrically more distant” and has less effect on the internal system. Any unmonitored portions of the internal system such as, lower voltage networks or unmonitored substations are also incorporated in the “external” system.

The internal system is solved by the State Estimator. A portion of the external system is usually retained intact and the rest is equivalenced. The extent of the equivalencing is usually dictated by memory limitations or availability of external data. In real-time mode the external network is solved by the state estimator in an one pass estimator or, by the external estimator in a two pass estimator which ensures consistency with the state estimation-based solution of the internal network. In the study mode the entire network (internal and external) is solved by the Dispatcher Power Flow. The equivalent network should be accurate enough to ensure the accuracy of other EMS applications. These include Dispatcher Power Flow, Security Analysis, Optimal Power flow and Dispatcher Training Simulator.

Traditionally the major thrust of research in external network modeling has focused on power flow analysis and security assessment applications. These efforts in developing and testing external network models are making a definite contribution to the rapid progress towards a good and reliable external network model. A comprehensive review of the most promising methodologies for external equivalents for security assessment is given in [1-3]. Extensive practical experience has indicated that Extended Ward based equivalents tend to perform in a satisfactory manner for most security analysis applications and tend to dominate the field [4]. They involve two basic steps; the construction of the equivalent network and the boundary matching [5]. The equivalent network is constructed using only external network data; the external operating-point data affects only the boundary injections. Since network data changes rather infrequently as compared to operating-point data that change from minute to minute, this type of equivalencing model is amenable to track, in real-time, changes of operating-point data through boundary matching. The lack of this separation property in other equivalencing models, such as REI [6-7], renders them ineffective to model changing operating conditions for some applications. However, published results [8] strongly indicate that REI equivalents can be useful for some applications. These results may encourage researchers to revisit the applicability of REI-based technology to power systems. In general, it seems, that the accuracy of the REI equivalents for security analysis depends on how the grouping of buses is performed [9-10]. REI-based variations have also been proposed [11-12] with some degree of success [13].

The developed methods mentioned above, based on the accumulated industry experience of the last decade, are accurate enough in ensuring that the performance of power flow and security analysis applications is not compromised. With some basic modifications, that are rather easy to implement, the power flow based reduced models can also be used effectively for Dispatcher Training Simulator applications.

However, if power flow based equivalencing methods are applied blindly for security control (in OPF applications for example) the results of the optimization would be unacceptable even for the base case, let alone the contingency cases. The problem has been identified in 114] but no solid solutions were offered. The errors result from poor accuracy of loss modeling in the equivalents and inability to monitor or enforce inequality constraints of the external system represented by the equivalent. If the losses in an equivalent are too low, the OPF algorithm will route too much flow through the equivalent. If the losses are too high, the effect will be opposite. If an equivalent is unconstrained, the OPF algorithm will circumvent binding inequalities by routing flows through the equivalent. To the extent that these effects occur, the solution of the network optimization is in error. Differences between the parameters of the equivalent and those of the external system will cause derivatives of the Lagrangian with respect to variables associated with the boundary buses to become nonzero. This inability by the well established boundary methods of the power flow based equivalents to nullify the OPF gradient at the boundary buses will cause changes in the OPF solution. Therefore a power flow based equivalent may potentially change, and sometimes substantially, the scope of an OPF problem. This calls for new approaches to external modeling that can be suitable for Optimal Power Flow applications.

PG&E has been investigating different approaches to develop optimization based external models over several years. This paper reports on the development. implementation and testing of the most promising methodology. It involves the reduction to the boundary buses of the composite Hessian/Jacobian matrix of the OPF. The resulting equivalent is an incremental model of the external system linearized at a selected base case solution point. In section II, the requirements for the optimization based external equivalents and the assumptions utilized are presented. The new proposed methodology suitable for optimization based external modeling is described in section III. In section IV, we present numerical testing results obtained by comparing the full network solution with the OPF and power flow based equivalent models using data from a large scale system. Major conclusions and the contributions of this work are summarized in section V.

II. OPF EXTERNAL MODELING REQUIREMENTS

In this section the assumptions made for the development of the OPF based external modeling techniques are presented along with a review of the complexities encountered in establishing a modeling framework that is suitable for optimization and security control. The requirements that need to be met for an acceptable OPF based external equivalent will also be discussed.

What sets apart an OPF from a power flow is the treatment of the control/constraints and the objective function. These same issues complicate greatly the process of developing equivalent modeling techniques suitable for security control. The control/constraints in the buffer zone, i.e., the portion of the neighboring utilities' system that is kept intact and fully modeled, can be handled the same way as in the internal system. However, it is not clear how the control/constraints of the external portion to be equivalenced should be represented. While it may be reasonable to assume that external operating constraints in real time are respected, it would be unrealistic to assume that the precise external control actions for achieving compliance are known. Furthermore, for realistic modeling a clear distinction between global and local controls needs to be made. Their treatment in external network modeling should be different as well. Global controls refer to the OPF control variables that participate in the optimization: local controls refer to the variables that are adjusted to satisfy a power flow type local objective other than the optimization objective. As an example, reactive generation control can be used either as a local or as a global control. When used as a local control, it is adjusted to regulate the terminal or remote voltage to a specified value. When used as a global control, it is adjusted to minimize the objective function and is only partially responsible for maintaining the desired bus voltage. The handling of the objective function also creates another level of complexity that needs to be dealt with. It is not clear whether we should have one external network model for all objectives or separate models for different objectives.

Given these complexities, we need to make some assumptions in applying the proposed method to compute the OPF based equivalents. In particular, we assume that: 1) a solved OPF base case of the full system is available; 2) the objective is a function of internal states and controls only; 3) the controls in the boundary areas are not active; 4) the inequality constraints of the external network are not enforced in the equivalent model; and *5)* the objective function for the reduced network remains the same as that of the full system. No other unusual assumptions need to be made in developing the proposed method including assumptions regarding the discreteness and the convexity of the OPF problem [18].

We feel that these assumptions are reasonable and mild enough, and do not pose significant difficulties in most of the applications. For example, solved OPF base cases are usually available from off-line computations and most of the objective functions used for OPF involve only internal system variables. We can also conveniently redefine the boundary buses if assumption 3) is not satisfied. As far as assumption 4) is concerned, we can not predict when external inequalities will become binding and the best we can do is to keep them constant. We can not model the external global controls since we can not change the control settings for the external companies. Lastly, assumption 5) simply reflects the fact that we don't know whether a single or different external equivalent models are needed for different OPF applications with different objective functions. These assumptions provide a reasonable foundation on which we can develop an OPF based external modeling methodology. This methodology will be able to produce an equivalent network model from a given full network model suitable for OPF applications. In this context, suitable" means that an external equivalent model has to satisfy the following requirements: 1) the external network is reduced and the internal network is kept intact; 2) the equivalent model satisfies the optimality conditions for the base case; 3) the equivalent model is compatible with the conventional power flow models; and 4) the incremental behavior of the full system and the equivalent model are approximately the same.

The first requirement simply indicates that the network size is significantly reduced to meet specific objectives of various nature. The second one specifies that the OPF solution using the equivalent model is the same as the full system solution. The desire for compatibility of the equivalent model with the conventional power flow model is motivated by practical reasons; if compatibility is not achieved the model would have limited use in practice. The last requirement is perhaps the most difficult to meet because of the random nature of the system changes and disturbances. In an EMS environment, the system conditions are constantly changing and the external network models should be able to model the external system for a wide range of system conditions. Since it is probably impossible to match the incremental behavior of the full network by an equivalent model exactly, we aim to produce an equivalent whose incremental behavior is as close as possible to that of the full system.

Although we are focusing on the development of methodologies for realistic OPF based external modeling techniques for EMS related functions, similar issues arise in planning studies. Detailed representation of a large size external system, when performing analyses over a multi-year planning horizon with a large number of contingencies greatly increases the size of the problem, thus leading to unrealistic CPU requirements. The proposed methodology could benefit off-line studies as well by ensuring the accuracy of the OPF applications and reducing the resource requirements needed for their use.

III. OPF BASED EXTERNAL MODELING METHODOLOGY

In this section, the proposed methodology is described. First the basic optimization framework is presented along with a comparison of the proposed method with the power flow based Extended Ward model. Then the major two tasks of the proposed method, i.e., incremental response modeling and boundary gradient matching are presented. Finally some important implementation issues are addressed.

A. Background

Assuming that the binding constraints are identified from a known base case OPF solution, then the OPF problem is formulated as follows:

$$\begin{array}{ll} & \text{Min } f(x) \\ \text{Subject to:} & \mathbf{C}(x)=0 \end{array} \qquad (1)$$

where f(x) is the objective function and **C**(x) is the vector of all constraint equations. The constraint equations should include the active and reactive power balance constraints, and all other binding constraints. Note that the non-binding inequality constraints are not included. The vector x of the solution variables includes bus voltage magnitudes and angles, active and reactive generations, transformer tap settings and other miscellaneous variables.

The Kuhn-Tucker optimality conditions require that the following equations be satisfied at the optimum solution:

$$\frac{\partial L}{\partial x}=0 \quad \frac{\partial L}{\partial l}=0 \qquad (2)$$

where $L = f(x) + \lambda^T C(x)$ is the Lagrangian function. The

Lagrange multipliers λ are readily available from the OPF solution. In case they are unavailable, they can be computed using least squares methods [15-16].

The development of the proposed methodology closely follows the basic steps of the Extended Ward method. The buses of the full network model are partitioned as internal, boundary, and external according to *a priori* knowledge of the full system. This knowledge is mainly based on the accumulated experience of the operators as well as the insight provided by analytical models. During the reduction process, the external buses are eliminated, the circuit elements of the boundary buses are modified and the internal system is kept intact.

However, there are two fundamental differences between the proposed methodology and the Extended Ward model. The first is that in the proposed methodology the Hessian/Jacobian matrix is used in the network reduction rather than the admittance matrix. The second is that the gradient optimality conditions as well as the power flow balance equations should be all satisfied when performing the boundary matching for OPF applications.

B. OPF Based Incremental Response Modeling

The incremental response of an OPF solution refers to how the state vector changes for a given parameter change in the neighborhood of the base case solution. The parameters of interest include bus loading, operating limits, equipment status, or any other parameters used for the OPF problem. For a given parameter change in the system, we can compute the solution vector changes by applying the chain rule to the optimality conditions in (2), which leads to:

$$H\begin{vmatrix} \dfrac{\partial x}{\partial e} \\ \dfrac{\partial l}{\partial e} \end{vmatrix} = Z, H = \begin{vmatrix} \dfrac{\partial^2 L}{\partial x^2} & \dfrac{\partial^2 L}{\partial x \partial l} \\ \dfrac{\partial^2 L}{\partial x \partial l} & \dfrac{\partial^2 L}{\partial l^2} \end{vmatrix} Z = \begin{vmatrix} \dfrac{\partial^2 L}{\partial x \partial e} \\ \dfrac{\partial^2 L}{\partial l \partial e} \end{vmatrix} \qquad (3)$$

where ε is the disturbance parameter, H is the composite Hessian/Jacobian matrix and Z is the gradient mismatch vector. By grouping together the variables of the same bus and arranging them based on their classification as external, boundary or internal buses, the matrix H and the vector Z in (3) can be rewritten in a partitioned form as:

$$H = \begin{vmatrix} H_{ee} & H_{eb} & 0 \\ H_{be} & H_{bb} & H_{bi} \\ 0 & H_{ib} & H_{ii} \end{vmatrix} \quad Z = \begin{vmatrix} Ze \\ Zb \\ Zi \end{vmatrix} \qquad (4)$$

In the above matrix, subscripts e, b and i represent the blocks associated with the external, boundary and internal buses respectively. The controls and constraints for the external system are modeled in the H_{ee} block. For external modeling purposes, the contingencies of interest are in the internal network. Consequently the elements of the external partition of the *Z* vector will be zero. Application of the Gauss elimination procedure to the H matrix will eliminate the coupling between external and boundary variables and the reduced H matrix will be in the following form:

$$H' = \begin{vmatrix} H_{ee} & H_{eb} & 0 \\ 0 & H'_{bb} & H_{bi} \\ 0 & H_{ib} & H_{ii} \end{vmatrix} \qquad (5)$$

$$H'_{bb} = H_{bb} - H_{be} H_{ee}^{-1} H'_{eb}$$

In the H' matrix the boundary and internal variables will be decoupled from the external variables. Using the internal and boundary portions of the H' matrix in (5), we can compute the solution vector changes for a given disturbance in the internal system. However, it is not feasible to present the H' as the basis for external equivalent models for OPF applications. There are two reasons that a parameter based model is needed: First, none of the commercially available OPF programs can accept the reduced Hessian/Jacobian matrix H' as input; Secondly the compatibility requirement is important for a reduced model since different applications in an EMS usually share a common network model. Therefore our objective next is to find the best approximation of the H' matrix using conventional circuit parameters in the reduced model.

In *(5)*, the H_{ii}, H_{ib} and H_{bi} blocks remain unchanged. Thus, we'll leave the internal network intact and use it as a starting point for the development of the equivalent model. Since only the boundary block in the H' matrix is changed, we'll concentrate on the modeling of the boundary buses. First we note that there are off-diagonal fill-in blocks in the H'_{bb} matrix that physically represent a coupling between the boundary buses through the external system. To model this coupling, we insert a branch between each pair of the boundary buses. The problem is to find the branch parameters to correctly model these fill-in blocks. Suppose we insert a branch between buses i and j with admittance g_{ij} and b_{ij} The newly added branch will create, in the matrix H'_{bb}, an off-diagonal block. H''_{bbij} which is a function of the unknown parameters g_{ij} and b_{ij}. It is not possible to match exactly the block H''_{bbij} with the original fill-in block H'_{bbij} since the number of equations to be satisfied is greater than the number of unknowns (g_{ij} and b_{ij}). The number of equations is related to the number of Hessian/Jacobian entries in the off-diagonal block H''_{bbij}. To resolve this problem, we propose to use a least squares method to find the best fit to the H’$_{bbij}$ block. Therefore we solve:

$$\mathrm{Min}\ \|\ H'_{bbij} - H''_{bbij}(p) \|$$
$$\text{with } p^T = [g_{ij}, b_{ij}]^T \quad \text{and } i \neq j$$

where H”$_{bbij}$ is the off-diagonal block due to the insertion of the new branch and H’$_{bbij}$ is the fill-in block. The branch parameters are adjusted so that the H”$_{bb}$ best approximates the fill-in block at the ij position of H’$_{bb}$ matrix. The above least squares solution is repeated for all off-diagonal, fill-in blocks that are generated due to the Gauss elimination in (5). At the end of this process, the admittances of all equivalent branches between boundary buses that best fit the reduced boundary blocks will be available.

To improve the reactive support of the equivalent model, we also create fictitious buses with radial connections to the boundary buses as in the Extended Ward model [5]. First we compute the Hessian blocks associated with the fictitious buses assuming that the initial radial parameters are known. These Hessian blocks include the diagonal blocks for the fictitious buses, off-diagonal blocks between the boundary buses and the fictitious buses, and additional contributions to the boundary buses. These blocks will determine the contributions of the radial connections of the fictitious buses to the boundary buses. Let H$_{fii}$ be the diagonal Hessian block of bus i that includes entries that are related to: a) the existing system, b) possible equivalent branches that connect bus i and other boundary buses (computed earlier), and c) the contribution of the radial connection of the fictitious bus to bus i. Again, it is not possible to match exactly the block H$_{fii}$(p) with the original diagonal block, H’$_{bbii}$, of the bus since the number of equations to be satisfied is greater than the number of unknowns (radial parameters). Therefore we solve the following least squares problem to best fit H’$_{bbij}$:

$$\mathrm{Min}\ \|\ H'_{bbii} - H_{fii}(p) \ \|$$
$$\text{with } p^T = [g_{fi}, b_{fi}]^T$$

where g_{fi} and b_{fi} are the conductance and susceptance of the radial branch connected at bus i. The above least squares solution is repeated for all boundary buses to compute the radial connections.

C. OPF Based Boundary Matching Modeling

The purpose of boundary matching is to adjust the reduced network in such a way that the base case OPF solution of the reduced model is identical to that of the full system. This requires that elements attached to the boundary buses be adjusted in a way that the optimality conditions for the reduced network are satisfied.

The optimality conditions for the internal buses are automatically satisfied since the internal system remains completely intact. The optimality conditions for the boundary buses are satisfied when the two gradient balance equations in (2) and the two power flow balance equations are met. However, the adjustments related to MVA loads and shunts, attached to the boundary buses and usually used with the Extended Ward method, have no effect on the bus voltage angle gradient. Therefore another element needs to be attached to the boundary buses to satisfy all necessary optimality conditions. A constant current load can play that role. The contributions of a constant current load to the voltage angle and magnitude gradients can be expressed as:

$$\frac{dLc}{dq} = V(-lgSinq + lhCosq)Ip + V(lgCosq + lhSinq)Iq$$
$$\frac{dLc}{dq} = (lgCosq + lhSinq)Ip + (lgSinq + lh\cos q)Iq$$
$$Lc = lgPc + lhQc \qquad (6)$$
$$Pc = V(IpCosq + IqSinq)$$
$$Qc = V(IpSinq + IqCosq)$$

Ip and Iq are the active and reactive constant current components, Pc and Qc are the MW and MVAR load components of the current load, Lc is the contribution of the constant current to the Lagrangian, λg and λ_h are the two power balance Lagrange multipliers and V and θ are the voltage magnitudes and angles respectively. Therefore, by attaching constant current loads to the boundary buses and adjusting the value of the two current components, the gradient balance equations can be satisfied. Hence, using MVA loads and constant current loads, the optimality conditions at the boundary buses can be exactly satisfied. To further improve the increment response of the reduced model, we have also used shunt elements, in addition to the constant current loads, for boundary matching. The following constrained least squares method is then solved for each boundary bus:

$$\mathrm{Min}\ \| H'_{bbii} - H''_{bbii}(p) \|$$

subject to:

$$\frac{\partial L(p)}{\partial v_i} = 0 \qquad \frac{\partial L(p)}{\partial q_i} = 0$$
$$\text{with } p^T = [g_{ii}, b_{ii}, I_{pi}, I_{qi}]^T$$

where H’$_{bbij}$ is the original diagonal block to be matched for bus i, H”$_{bbij}$(p) is the diagonal block that include components from all previous sequential additions described in this section and the contributions from the shunt and constant current loads, g_{ii} and b_{ii} are the shunt admittance parameters, and Ipi and Iqi are the active and reactive constant current components at bus i.

The computational overhead for solving the large number of the least square problems is very limited. The dimension of these problems associated with the proposed method ranges from 2 to 4. In the test results shown in the next section, the least squares solution takes less than 6 percent of the total CPU time.

D. Implementation Issues

There are several practical considerations that need to be taken in account in building an equivalent model for use in the EMS environment. The creation of buffer zone, the frequency of model update, the validity of the model for different conditions and different optimization objectives, and sparsity considerations are only a few of the issues that need to be properly handled. These issues will be discussed next.

Special attention should be given to the modeling of the beyond-the-boundary internal power system that is kept intact (buffer zone). Topology information and regulated voltage levels for the buffer zone is of critical importance. Accurate information regarding interchange schedules and third party transactions is also very important. Critical lines, buses or other facilities that are known to be affected by internal control actions should also be included in the buffer zone. Based on extensive testing, failure to include these special facilities may lead to poor solutions, or even divergence of the network applications. Utilities are encouraged to perform their own studies to determine the optimal buffer zone to ensure the accuracy of the network applications. Engineering judgment and experience along with analytical studies should be used for this purpose.

In general, the frequency of model update depends on the specific application requirements, system configuration, availability of data and magnitude of the boundary mismatches. The larger the mismatches, the greater, in general, the need for an update. Most of the power flow based models in practice are produced off-line, are maintained manually and are updated when a new EMS build comes on-line. Until further experience is accumulated, we recommend to follow the same guidelines for models produced with the proposed methodology. Alternatively, a "superstructure" that consists of pre-stored equivalent models that are embedded in the EMS network model can be developed. These predefined models will be designated to have status corrections and impedances dynamically changed by an on-line External Equivalence Calculation (EEC) function that will become part of the Real-Time Sequence functions. This added capability will offset, to some extent, the limitations imposed by the inability to reconstruct on-line the network model structure. On-line construction of OPF-based reduced models is further complicated by the need to derive and match, in real-time, the gradient optimality conditions. The selection of the appropriate model to be used at any given time by the EEC function should be made based on engineering judgment and current operational guidelines.

For the implementation of the proposed methodology, we used the most common objective of minimizing production costs. In principle, the proposed methodology can also be applied to different OPF applications that involve different objectives such as losses, reactive requirements, security margins, etc. Dependence of the objective function from the internal variables only is the only limitation of the proposed methodology in that regard. However, further modifications may be required to adapt the methodology to handle specific requirements related to these applications. These modifications may include the adjustment of the buffer zone, inclusion of special external facilities in the buffer zone and tuning of the frequency of the model update. Specifically, for wheeling studies, the networks of all parties involved in a specific transaction should be included. In practice, and with the current level of wheeling, simplified network representations of the external companies involved in a transaction are adequate. However, the level of network reduction may need to be modified as the number and the magnitude of wheeling transactions increase as a result of emerging regulatory reforms in various part of the country.

Network sparsity issues should also be considered when reduction is needed. With the proposed methodology, the fill-ins generated by the reduction of the Hessian/Jacobian matrix have the same pattern as in the traditional equivalent modeling method where the admittance matrix is reduced. That is. whenever a fill-in between two buses is created by the admittance based reduction method, the same fill-in will occur with the proposed method. Therefore techniques such as the adaptive reduction approach in [17], that have been successfully applied with conventional reduction methodologies, can also be applied equally well with the proposed method.

IV. TEST RESULTS

Based on the methodology described in section III, we have implemented a computer program that can compute external equivalent models from base case OPF solutions of a full network. We utilized this program to produce an OPF based equivalent model that will be used as a basis for our testing below. We have also developed a conventional power flow equivalent model based on the Extended Ward method. This model has been extensively tested over a wide range of operating conditions and has been integrated with the EMS network model for more than three years. Based on actual experience, it has been found to accurately represent the responses of external system to the PG&E system for security analysis. The OPF and power flow (PF) based reduced equivalent models have been compared with the full system model, i.e., the full model was used as a reference in all tests. Both reduced models have identical boundary and internal buses. In addition, both the PF and OPF based equivalent models have fictitious PV buses attached to the boundary buses for improving the VAR response during contingencies.

The testing system comprises of the full WSCC (Western System Coordination Council) network. The characteristics of the system are presented in Table 1. The objective function for the OPF is the total production cost for the dispatchable units of the internal system. In each test, we compared the solution variables of the internal system (i.e., the buses in the buffer zone were excluded) and listed the standard deviation of the errors (El) and largest errors (E2) for each of the following variables: objective function (if applicable), voltages, generation outputs and branch flows. The CPU time for the equivalencing program to execute is about 220 seconds on an Apollo 400 series workstation. In this test, there are 160 of fill-in blocks generated in the Hessian/Jacobian matrix by the reduction process. The least squares solution and the boundary matching function took less than 12 seconds. Most of the computational effort is spent on the matrix factorization and reduction.

The base case OPF test results are listed in Table 2. In this test, the equivalent models are solved by the OPF with no contingencies. The base case solution of the OPF based equivalent is very close to the full system solution. The small differences are due to numerical truncations. The errors for the PF based equivalent model, on the other hand, are much larger, especially for the reactive power generation and flows. These errors of the PF equivalent exist because the optimality conditions at the boundary buses are not satisfied for the PF equivalent model. The base case power flow solutions of the three models are all the same for the internal system and are not presented here.

Table 3 presents the results of the comparison of the full OPF solution with the OPF based and PF based equivalent model solutions for an outage of a unit of 475 MW. As can be seen, the OPF equivalent model is very accurate in solving the active power portion of the OPF solution. The standard deviation of the voltage magnitude error was 0.02 kv, thus 95.4 percent of the voltage solutions have errors within 0.04 kv. The errors for the PF equivalent model are much larger, especially for the reactive power generation and flows. For example, the largest MVAR flow error for the OPF equivalent model is 2.52 MVAR and for the PF equivalent is 44.19 MVAR.

Table 4 presents the result of the comparison of the full OPF solution with the OPF based and PF based equivalent model solutions for a 500kv transmission line outage carrying 844 MW. As can be seen again, the OPF equivalent model is very accurate in solving the active portion of the OPF solution. The relative poor performance for the reactive variables can be due to the week coupling of these variables to the cost objective function. The errors for the PF equivalent model are much larger especially for the reactive power

Table 1: Test Case Description

System Description	WSCC
Number of buses	2436
Number of generators	657
Number of branches	3978
Number of internal buses	646
Number of boundary buses	31
Number of buffer zone buses	77
Total system load in MW	106680
Total generation in MW	109754
Total number of companies	23

Table 2: OPF Solutions for the Base Case

Model	OPF Equivalent		PF Equivalent	
Error	El	E2	El	E2
Cost Objective		0.154		4.780
Voltage (kv)	0.020	0.183	0.285	2.076
MW Gen.	0.007	0.083	0.137	0.936
MVAR Gen.	0.298	3.340	2.117	24.300
MW Flow	0.017	0.197	0.225	2.556
MVAR Flow	0.244	3.400	3.020	43.19

El: Standard Deviation E2: Maximum Error

Table 3: OPF Solutions for an Unit Outage

Model	OPF Equivalent		PF Equivalent	
Error	E1	E2	E3	E4
Cost Objective		0.400		5.180
Voltage (kv)	0.023	0.169	0.257	1.977
MW Gen.	0.016	0.255	0.240	3.190
MVAR Gen.	0.225	2.520	2.753	25.43
MW Flow	0.351	2.200	1.102	6.180
MYAR Flow	0.231	2.520	3.040	44.19

Table 4: OPF Solutions for a Branch

Model	OPF Equivalent		PF Equivalent	
Error	El	E2	El	E2
Cost Objective		1.64		9.430
Voltage (kv)	0.074	0.380	0.316	1.556
MW Gen.	0.036	0.519	0.274	1.681
MVAR Gen.	0.641	4.134	3.310	15.54
MW Flow	2.662	20.10	6.630	59.62
MVAR Flow	1.025	8.850	4.550	49.72

Table 5: PF Solutions for an Unit Outage

Model	OPF Equivalent		PF Equivalent	
Error	El	E2	El	E2
Voltage (kv)	0.110	0.664	0.160	0.762
MW Gen.		0.291		1.900
MVAR Gen.	0.546	4.750	1.160	10.56
MW Flow	1.660	10.85	1.790	13.20
MVAR Flow	0.926	8.900	1.530	16.08

Table 6: PF Solutions for a Branch Outage

Model	OPF Equivalent		PF Equivalent	
Error	El	E2	El	E2
Voltage (kv)	0.250	1.250	0.280	1.370
MW Gen.		1.170		1.900
MVAR Gen.	0.946	10.76	1.500	10.50
MW Flow	2.740	20.76	2.650	18.46
MVAR Flow	1.480	12.16	2.170	17.00

variables. For example, the largest MVAR flow error for the OPF equivalent model is 8.85 MVAR and for the PF equivalent model is 49.72 MVAR. Other similar tests, with smaller buffer zones, not presented here, strongly indicate that the MVAR flow errors for the PF equivalent model can be as high as 100 MVAR. On the other hand, the errors for the OPF equivalent models are in the same level with the ones presented in this paper.

The above two outages were also applied to the two equivalent models and were solved using a power flow program. The results are presented in Tables 5 and 6. A can be seen the results are similar, even though the OPF equivalent model outperforms the PF equivalent model in most cases by a slight margin. Numerous other test results, not presented here, are consistent with the above conclusion. This observation indicates that we can use OPF based equivalent models not just for OPF applications but for other network applications such as power flow analysis and security assessment without loss of accuracy. This is a conclusion of significant importance because of its implication for database maintenance efforts. Based on practical experience, database updating and maintenance have proven to be a complicated and time consuming task for most systems. Ability to have one database to maintain instead of more (depending on the EMS applications) is of great significance. Based on the above results, our proposed methodology could be used to provide equivalent models for security analysis and security control without creating an additional burden in maintaining and updating the EMS database.

V. CONCLUSIONS

Traditionally work on external network modeling has been focused on developing methodologies that are able to produce external network equivalents suitable for power flow analysis and security assessment applications. Based on the accumulated industry experience of the last decade, some of these methods are accurate enough in ensuring that the performance of power flow and security analysis applications is not compromised. However, these methods cannot be used for Optimal Power Flow applications. They can potentially change, and sometimes substantially, the scope of an OPF problem. The errors result from poor accuracy of loss modeling in the equivalents and inability to satisfy the optimality conditions at boundary buses.

In this paper, we present a new methodology for external network modeling that is suitable for optimization based applications. We view this development as an important milestone in expanding the scope and enhancing the capability of current external network modeling methodologies to support security analysis as well as security control applications in an Energy Management System. The proposed methodology has the following unique features:
a) ability to produce equivalent models that match exactly the OPF solution of the original full network for the base case by utilizing the gradient boundary matching technique, b) ability to produce equivalent models whose incremental response is very close to that of the full system by reducing to the boundary buses the composite Hessian/Jacobian matrix of the OPF problem, and c) ability to compute models that are fully compatible with those required by other EMS network applications.

The proposed methodology has been extensively tested under a wide variety of operating conditions using data from a large scale system. The tests presented in the paper, and other numerous tests not presented here, strongly indicate that the proposed methodology outperforms the Extended Ward method in producing equivalents suitable for security control. The tests also demonstrate that the proposed methodology could be used to produce equivalents not just for OPF applications but also for other network applications, such as power flow and security analysis without loss of accuracy. This is an important contribution because it implies that the proposed methodology can be used for security analysis and security control without creating an additional burden in maintaining and updating the EMS database.

VI. REFERENCES

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VII. BIOGPAPHIES

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