# Cost Allocation in Electric Power Networks using Cooperative Game Theory

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Abstract: This paper deals with the problem of allocating common costs in electric power networks using principles from cooperative game theory. The paper discusses how these principles offer an extension to the earlier "use of system" based criteria for allocating network fixed costs. The principles illustrated in the paper have been used in many different settings and can be applied to address issues such as transmission common cost allocation and generating unit joint cost allocation. *Deregulation/Pricing, Shapley Value, Core Allocations.*

## I. Introduction

Electric utilities are being restructured in many parts of the world to facilitate greater competition in the supply of electrical energy with the hope of lowering the rates to all consumers. This restructuring process is a result of many technological and legislative factors. One of the most important factors includes a decline in the levels of economies of scale in generation. Important legislative factors in the United States include the Public Utility Regulatory Policies Act (PURPA) of 1978 and the National Energy Policy Act (EPAct) of 1992. Restructuring initiatives at the state level (e.g. the California Public Utilities Commission initiative of April 20, 1994) and recent policy statements by the Federal Energy Regulatory Commission (FERC) (e.g. the Notice of Proposed Rulemaking (NOPR) issued on March 29, 1995, also known as the Mega-NOPR) have also been significant in accelerating the pace of electric utility restructuring.

The emerging utility structure, even though years away from full implementation, will probably pave the way for vertical disintegration of the existing structure with the transmission and distribution functions being separated from the generation function. Considerable attention in the restructuring debate has been devoted to costing and pricing issues for unbundled transmission services [4,8,11,14]. This paper addresses cost allocation principles that are applicable to many of these issues and applies them to the allocation of network fixed costs.

Section II of the paper discusses the costs of unbundled electric services and section III discusses the allocation of network fixed costs based on network usage. Section IV provides an overview of essential concepts from cooperative game theory. Section V presents illustrative examples. The conclusions are presented in section VI.

## II. Costs of Unbundled Services

Under the vertically integrated structure of electric utilities, transmission and generation costs were bundled together in the prices for these services. Both transmission and generation were regulated and without any significant competition. To simulate prices that might exist in a competitive environment, much attention was devoted to the calculation of marginal costs of electricity. It is well known that for cost functions exhibiting increasing returns to scale, the marginal costs are lower than average costs [17]. Thus, prices based on marginal costs usually involved adjustments in the form of adders or multipliers[1]. The situation has changed somewhat in the present paradigm of competitive generation that involves the supply of a number of unbundled services. It is assumed that basic energy prices will be set by the market[2]. The supply of ancillary services[3] such as transmission losses and transmission congestion management has been the focus of intense discussions as a part of the debate about the overall market structure. Ironically, the use of prices based on marginal costs

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[1] Theoretically optimal approach here is known as Ramsey pricing and is based on knowledge of customer elasticities [9].

[2] Whether or not these prices will actually tend to equal the short run marginal costs is still a subject of debate.

[3] The term *ancillary services* is most often used to refer to generation related services such as reactive support, load following and reserve capacity [6]. The ideas proposed in this paper may also be used to allocate joint costs of generating units among these services.

for transmission losses and congestion results in revenues that exceed the average cost of providing them [4, 11].

The issue of how the embedded costs of the network will be shared by users has been central in the current debate. A common theme among previous approaches to this problem has involved the notion of equity that requires users to pay for accessing the transmission network according to some measure of usage[4]. Among the different criteria that can be used to evaluate and compare the various measures of usage are the issues of *accuracy* and *stability*. It is generally accepted that power-flow based measures of usage such as the MW-mile and its variations are more accurate than measures based on contract path definitions. The issue of stability is related to the question of fairness [8]. From a cost allocation and equity perspective, stability is an equally if not more important criterion and is the motivation for the ideas expressed in this paper.

## III. Usage Based Cost Allocation

Different methods have been proposed for allocating network costs among users based on usage [3,5,7,8,16]. For the purposes of this paper, a stylized representation of the problem is sufficient. The basic idea is to allocate to each transaction *t* (between a buyer and a seller), a cost C(t) where

$$C_t = \frac{Usage_t}{\sum_{t \in t} Usage_t} \quad \text{Network Cost}$$

where τ is the set of all transactions. This suggests a simple method for allocating an appropriate fraction of network costs to each transaction. As a practical matter, the denominator in the above equation can be based on capacities of the facilities involved in the transaction if the set τ is not known a-priori. The different measures of usage based on MW demand can be classified as:

a) Non power flow methods: Among the simplest measures is the so called *postage stamp* method that considers only the magnitude of the transaction in MWs. It is a very approximate measure as it completely ignores locational and distance aspects. Another set of methods considers not only the magnitude of transactions but also the distances in terms of the *contract path*. These methods are also an approximation as they ignore the physical reality that all facilities and not just the ones in the contract path are involved in a transaction.

b) Power flow methods: A more accurate measure of usage can be obtained by performing a "power flow" solution to calculate the "MW-miles" involved in a transaction [16,19]. Two such usage measures are the *"weighted algebraic sum"* and the *"weighted sum of absolute values"* of flows over all lines involved in a transaction [3,7,8]. Both these criteria are equally accurate as they are both based on the actual flows that occur m the system. However as illustrated later in the paper, they can result in very different cost allocations. The question then becomes one of deciding which criterion is more "fair". In order to examine the fairness of a set of equally accurate criteria, we need to introduce some ideas from cooperative game theory as described in the next section.

## IV. Cost Allocation using Cooperative Game Theory

Cost allocation involves partitioning a cost among a set of objects. Two major classes of the cost allocation problem involve joint and *common* costs [2]. The term joint cost applies to a setting where costs of production are a non-separable function of a set of products. The non-separability of the cost function and the joint production results in cost savings that are often characterized as *economies of scope* [1]. The term common cost apples to settings in which the production cost is defined over a single product that is used by multiple users. Common production is undertaken to realize cost savings due to *economies of scale*. The problem of allocating costs of transmission systems among different users is a common cost problem. Whereas joint cost allocation formulations emphasize output decision incentives, common cost allocations emphasize incentives for users to participate in the common provision of the service. Both types of problems lend themselves well for the application of game theory. This paper addresses the common cost allocation problem.

Game theory may be classified into two areas- *cooperative* and *non-cooperative*. This paper applies principles from cooperative game theory to the network cost allocation problem. Cooperative games can be applied to allocation problems and the various solutions proposed for such games can be interpreted as alternative solutions to an allocation problem. Let *N = {1,…,n}* represent a set of users (or outputs)[5] of a facility (or firm). Each user is served at some level that is greater than or equal to zero. The costs are specified by a common cost function *c(S)* defined over every subset *S Í N* of users[6] . The cost of serving no-one (or producing nothing) is assumed to be zero *c(Æ)=0*. A cost allocation method will produce an outcome that associates an allocation $(x_1,..., x_2)$ with all cost functions *c* defined on *N*, i.e.,

[4] Usage," in this paper refers to *use of the system* rather than use of energy. Consequently, it implies a demand charge rather than an energy charge.

[5] Since, this paper is concerned with common costs, we will use the term "users" (or "transactions") instead of "outputs". Both cases can be represented by using the term "players".

[6] Subsets of users are also known as *coalitions*

$$\Omega(c) = (x_1,..., x_n) \in \Re^N \text{ and } \sum_{i=1}^{N} x_i = c(N) \quad (1)$$

where $x_i$ is the cost allocated to user *i.* In this section, we discuss how cooperative game theory can be used to find suitable cost allocations. The foundations of cooperative game theory can be traced back to the work of Von Neumann and Morgenstern [18]. However, many of the essential ideas such as the "core", were developed later by others including Shapley *[15].* These game theoretic ideas were preceded by work done on cost allocation. The best known example is the Tennessee Valley Authority Act of 1933 [10,13]. This Act dealt with the issue of allocating the costs of the TVA project among its different purposes, e.g., navigation, flood control and electric power. The concepts devised to deal with this problem foreshadow developments in game theory. One such concept that is widely used in the cost allocation literature is the so-called *stand alone* test that requires that no user be allocated a cost that is greater than what it would cost to serve that user alone (i.e., its *stand alone cost* or its alternative cost), i.e.:

$$x(S) \leq c(S), \quad (2)$$

where $x(S) = \sum_{i\in S} x_i$ and *c(S)* is the common cost function for the *coalition S.* A related principle involves *separable cost* (also called the incremental or marginal cost) of including a particular user and requires that no user be allocated a cost less than its separable cost. i.e.:

$$x(S) \geq c(N) - c(N-S)\ \forall\ S \subseteq N. \quad (3)$$

A third requirement is the costs be allocated exactly, i.e.:

$$x(N)=c(N) \quad (4)$$

This condition is sometimes referred to as "pareto optimality" or the *"break-even"* requirement. The inequalities represented by the stand-alone test (2) are referred to as *"coalition rationality."* A subset of the inequalities in (2) that deals with the case of individual users (i.e. S is a singleton), is referred to as *"individual rationality."* Cost allocations that are pareto optimal and individually rational are called *"imputations."* It is easy to represent the set of imputations for the case of 3 users (i.e., 3 player cooperative games) graphically as shown in Figure 1 .The set of imputations is a subset of the 2 dimensional simplex[7] in $\Re^3$ The set of imputations can be represented more concisely by projecting the 2-dimensional simplex onto a plane

from a suitable viewpoint. This is shown in Figure 2 in which each vertex is labeled as one of the users. The cost allocated to each output is measured from the baseline opposite to the vertex corresponding to the user.

The set of allocations that satisfy the stand-alone conditions (2) and the break-even condition (4) represent solutions that lie in the *"core"* of the cost allocation game. The core is always a subset of the set of imputations. The bounds used to generate the core are based on the stand alone costs. However, they may also be generated by using the separable costs (3). Stand-alone costs represent upper bounds on allocations while separable costs represent lower bounds on cost allocations. There is an upper and a lower bound on each user and on each set of users (i.e. coalitions). Upper and lower bounds taken together with the break-even condition result in a set of redundant conditions. In case of three users, the separable cost of one user is a lower bound on the cost allocated to that user. It is equivalent to the upper bound defined by the stand alone cost for the other two users when combined with the break-even condition. Conversely, the upper bound defined on one user is equivalent to the lower bound on the other two users. Thus, the core may be defined by either using upper bounds (stand-alone costs) or lower bounds (separable costs). *Cost allocations that lie in the core can be identified with the absence of cross-subsidization.* The problem with the core is that it can represent a very large set of allocations or none at all (i.e., the core may be empty). We will focus on the problem of selecting a unique solution from the set defined by the core.

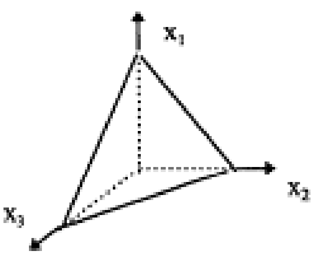



**Figure 1:** *Set of pareto optimal cost allocations for a 3 player game*

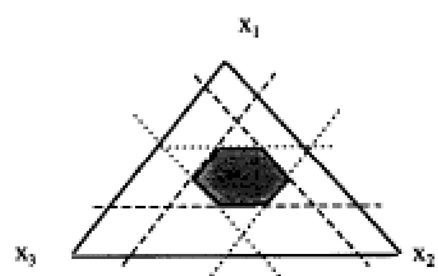



**Figure 2:** *The core of a 3 player game. The short dashed lines represent upper bounds on allocations to each output while the long dashed lines represent lower bounds on allocations to each player. Each vertex represents a solution in which the entire cost is allocated to that player alone.*

[7] A simplex in $\Re^n$ is a set of (n+1) equidistant points. Every point in the region enclosed by the simplex has the property that the sum of its coordinates is constant.

One approach to the problem of selecting a unique core allocation is based on the notion of making the least "well-off' coalition as "well-off' as possible. The notion of the "worth" of a coalition can be described by saying that coalition A is "better-off' or is worth more than coalition B if:

$$c(A) - x(A) > c(B) - x(B) \qquad (5)$$

where, recall that for any coalition, *x(S)* is defined as $\sum_{i \in S} x_i$ . The worth of a coalition S may be defined as the quantity *e(x,S) = c(S) - x(S)* which is in effect the savings realized by that coalition. The problem of maximizing the minimum worth *e(x,S)* may be written as the following linear program:

$$\max \alpha$$

$$\text{subject to } e(x,S) \geq a \;\; \forall\, S \neq \emptyset, N \qquad (6)$$

$$\sum_N x_i = c(N) \qquad (7)$$

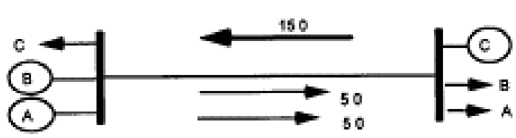
In general, this problem may not have a unique solution (e.g. the second example in the next section). The "Nucleolus" is a unique solution to the above problem that minimizes the worth *e* lexicographically, i.e., a solution for which the next smallest value of *e* is as large as possible and is obtained over as few sets as possible [20].

## V. Allocation of Network Costs: Illustrative Examples

In this section, we apply the concepts presented in the previous section to allocate network costs to various transactions that occur simultaneously. Two types of effects that can occur to create economies of scale (i.e., benefits of cooperation) are discussed. Other formulations are also possible.

*Example 1 (Counterflows):* One of the factors that contributes to economies of scale in terms of usage among transactions is the phenomenon of counterflow. These effects can be quite subtle in some network configurations. We consider a simple hypothetical example involving three transactions between two locations across a single transmission line as shown in Figure 3. Transactions A and B are 50 MW each and occur in a direction opposite to transaction C which is 150 MW. It is further assumed that in order to recover network costs, a price of $λ/MW (say 100) is applied[8]. Since the net loading is 50 MW, the total costs to be recovered are $5000.

Figure 3: *Three transactions over a single transmission line.*




[8] It is assumed that based on expected net loadings, line capacities and revenue requirements, it is possible to establish a value for $\lambda$.

The proposed method involves identifying the stand-alone costs of the transactions. The stand-alone cost of any transaction or set of transactions is the cost due to that transaction (or set of transactions) alone. The separable cost of any transaction S is its incremental cost defined as *c(N) - c(N*S) These costs are described in Table 1[9].

Table 1: *Stand-alone and separable costs for different subsets (coalitions) of transactions.*

| | Stand Alone Cost | Separable Cost |
|---|---|---|
| c{A} | 5000 | -5000 |
| c{B} | 5000 | -5000 |
| c{C} | 15,000 | -5000 |
| c{A,B} | 10,000 | -10,000 |
| c{B,C} | 10,000 | 0 |
| c{C,A} | 10,000 | 0 |
| c{A,B,C} | 5000 | 5000 |

The linear inequalities represented by either the separable costs or the stand-alone costs taken together with the break-even requirement are sufficient to define a set of subsidy free equitable cost allocations. The Nucleolus represents one such solution as explained in the previous section. The Shapley value is an alternative to the Nucleolus that assigns to each output its average contribution to a random subset of outputs 115]. The Shapley value is described in the appendix of the paper and is sometimes preferable in common cost problems as it results in *monotonic allocations.* A monotonic allocation implies that the share allocated to a coalition cannot increase if the stand-alone costs of that coalition actually decrease. Consequently, monotonicity of an allocation can have important consequences in terms of providing appropriate incentives. Table 2 illustrates the cost allocation solutions using the Nucleolus and the Shapley value along with two conventional schemes. In this case both the cooperative game theory solutions allocate all the $5000 to transaction C. In conventional solution (a), the *sum of absolute values* is used to apply the usage based method defined in section II of the paper. This allocates $1000 each to transactions A and B and assigns $3000 to transaction C. Such a scheme provides an incentive for either A or B to combine with C and decrease their allocated portion at the expense of the remaining transaction. For example, if A and C get together, their allocation would be $3333.3 and that of B would be $1666.6. Thus, the method is not stable.

Conventional solution (b) uses the *algebraic sum* to calculate the allocations.

[9] The cost function described here is not a *monotonic cost function*, i.e., the costs do not necessarily increase as the number of users increases.

**Table 2:** *Cost allocation comparison*

| | Transaction A | Transaction B | Transaction C |
|---|---|---|---|
| Nucleolus | 0 | 0 | 5000 |
| Shapley Value | 0 | 0 | 5000 |
| Conventional (a) | 1000 | 1000 | 3000 |
| Conventional (b) | -5000 | -5000 | -15000 |

*Example 2 (Non-coincident peaks):* If the temporal characteristics of transactions are taken into account, it becomes obvious that economies of scale can also occur due to reasons other than counterflows, e.g., the temporal complementarity or non-coincident peaks of transactions [12]. This is shown in Figure 4 where three transactions A, B and C are tracked over the three different time periods (e.g. seasons). The peak demand for the three transactions is 60, 30 and 50 MW respectively. These three peaks occur in different time periods (in time-period III for A, in I and II for B and in I for C). The net demand due to all three transactions taken together is 100, 80 and 90 MW in the three time-periods respectively. The stand-alone and separable costs for the three transactions (assuming a price of *\$50/MW)* are shown in Table 3.

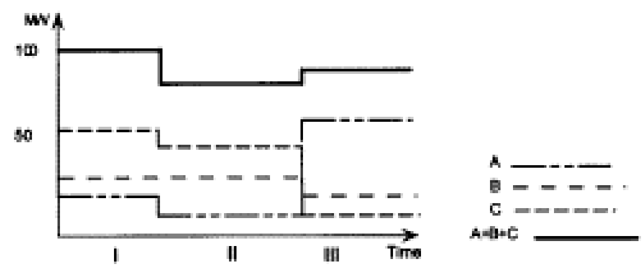



Figure 4: *Temporal complementarity of transactions.*

**Table 3:** *Stand-alone and separable costs for different subsets (coalitions) of transactions.*

| | Stand Alone Cost | Separable Cost |
|---|---|---|
| c{A} | 3000 | 1000 |
| c{B} | 1500 | 1500 |
| c{C} | 2500 | 1000 |
| c{A,B} | 4000 | 2500 |
| c{B,C} | 4000 | 2000 |
| c{C,A} | 3500 | 3500 |
| c{A,B,C} | 5000 | 5000 |

Table 4 shows the resulting allocations according to cooperative game theory based and conventional criteria. The *core* of this problem is a line segment given by $x_b = 1500$, $x_a + x_c = 3500$, $1000 <= x_a, x_c < 2500$. The Nucleolus is the midpoint of that line. The Shapley Value does not lie in the core for this example. This is a consequence of the nature of the cost function which happens to be non-concave[10]. The conventional solution allocates costs according to the individual peaks alone.

**Table 4:** *Cost allocation comparison*

| | Transaction A | Transaction B | Transaction C |
|---|---|---|---|
| Nucleolus | 1750 | 1500 | 1750 |
| Shapley Value | 1917 | 1416 | 1667 |
| Conventional | 2143 | 1071 | 1786 |

Both the Shapley value and the Nucleolus were used to compute solutions in small test systems. From a computational perspective such solutions require additional effort compared to the earlier usage based criteria. This is a consequence of the effort needed to calculate the stand-alone costs of all coalitions of transactions. In addition, the Shapley value computation involves solving a combinatorial problem.

## VI. Conclusion

The restructuring of the industry from monopoly regulation to a free-choice market has touched off a debate over how to cost and price unbundled transmission and generation services. Central to this debate is the cost allocation problem for products and services produced by the generation and transmission system. This paper focused on the problem of allocating network fixed costs among various network users. It described the notion of equity in cost-allocation from a game-theoretic perspective. Two different solutions, the Nucleolus and the Shapley value were discussed. The results presented in the paper suggest that cooperative game theory principles provide a good tool for addressing cost allocation problems. However, the computational challenges in applying these ideas to large scale problems remain to be addressed.

## Disclaimer

The material presented in this paper does not necessarily reflect the views of the Pacific Gas and Electric Company or Iowa State University.

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[10] The core of a concave cost function is non-empty and contains the Shapley value [15]. Allocations produced by the Shapley value may not in general lie in the core but are monotonic [21].

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## Appendix

**Shapley Value**

A central idea in cost allocation is the principle of allocating to each player a share of costs that is at least equal to that player’s marginal or separable cost. Taken over all possible coalitions and together with the break even requirement, this is defined as the “core” of the game. The separable cost for a player or coalition S is defined as *c(N) - c(N-S)*, i.e. it is defined with respect to the “grand coalition” consisting of all the players in the game. More generally, the separable cost for a player *i* may be defined over all possible coalitions including the empty coalition as c(S) - *c(S-i)* where coalition S is a subset of *N* (not necessarily a proper subset).

The Shapley value is the average value of such separable costs for a given player taken over all possible coalitions that contain player *i* and recognizing that there is equal probability of the occurrence of a given sized coalition. Consider player *i* in a three player game. The four coalitions (of three different sizes) containing player *i* are *(1), (1,2), (1,3)* and *(1,2,3).* The probability of occurrence of these coalitions is *1/3, 1/6, 1/6* and *1/3* respectively. The separable costs for player *1* are *c{1}, c{1,2}* - *c{2}, c{1,3}* c{3} and c{1,2,3} - c{2,3} in each of these three cases. Denoting these as $S_a$, $S_b$, $S_c$ and $S_d$ respectively, the Shapley value allocation for player i is ($S_a$/3) + ($S_b$/6) + ($S_c$/6) + ($S_d$/3). The allocations for players 2 and 3 are calculated similarly. For an N player game, the allocation to player using the Shapley value may be written as

$$x_i = \sum_{S \subseteq N} \frac{(N - |S|)!(S-1)!}{N!} c(S) - c(S-i)$$

where |S| denotes the size of coalition S.

<u>*Monotonicity:*</u> An important property satisfied by Shapley value allocations is allocation monotonicity [21]. More specifically this relates to the question of whether the allocation for a coalition decreases or increases for a corresponding change in costs. In general, it is possible that the allocation may increase even if the costs for the coalition decrease. This is true for all “core allocations” such as the Nucleolus.