Pricing Energy and Ancillary Services in Integrated Market Systems by an Optimal Power Flow

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Abstract—A detailed AC OPF-based formulation for procuring, pricing, and settling energy and ancillary service in simultaneous auctions by integrated market systems is presented. The paper provides clear definitions of Locational Marginal Prices for energy and Ancillary Service Marginal Prices in terms of Lagrange multipliers. The characteristics of the prices are analyzed especially when economic substitution among ancillary services is required. The paper also evaluates the conditions under which opportunity costs are incurred to units that provide ancillary services. It is particularly shown that the intuitive belief that the provision of regulation down service does not incur opportunity cost to the provider, in general, is not true.

Index Terms—Power system economics, optimal power flow, deregulation, locational marginal price, ancillary service, transmission congestion, transmission losses, opportunity cost.

I. INTRODUCTION

COMPETITIVE energy markets are instituted around the world and electric supply industries are restructured to compete in the new emerging markets. In general, two extreme forms of market auctions exist for trading of various energy products and services. Their difference stems from choosing between tighter coordination and greater reliance on private markets. Certain hybrid versions that claim to obtain the best of both market forms are also beginning to emerge.

In the first form of auction used in market systems, called unbundled systems, market products are procured sequentially through central auctions managed by the ISO/RTO. The initial market is the energy market, followed by a transmission market to manage congestions, followed by a market for Ancillary Services (A/S) to conform to mandated reliability criteria. The forward markets (on a day-ahead and hour-ahead basis) are followed by a real time market in which the ISO/RTO uses A/S energy and supplemental energy offers to balance the system in real time. Participation in each market is voluntary, so that traders can move freely from one market to another to arbitrage price differences between the markets. Proponents of these auctions claim that the voluntary nature of market participation allows the efficiencies provided by an “optimized” pool to be captured without having to deal with all the problems associated with complex optimization software tools. [1]. Examples of unbundled systems are in Australia, Scandinavia, California 1998-2000, and Texas, as well as in Britain’s new system that began operation in 2001.

In the second form of auctions used in market systems, called integrated systems, market products are procured simultaneously through central auctions. The incentive for developing integrated systems is to realize gains from tight coordination in daily operations, while strengthening system reliability. The basic argument for integrated systems is that optimization is necessary to minimize the total costs of coordinating generation, transmission and reserves to meet demand and ensure reliability. Proponents of integrated systems claim that the resulting pricing is superior in the sense that the shadow prices derived from the constrained optimization accurately reflects the system-wide opportunity costs of scarce resources, both inter-temporally and spatially. Examples of integrated systems are in Britain 1989-2001 and in the US, in New York, New England and PJM. Current experience from operating energy markets seems to give credence to the claim that practical unbundled systems, as currently implemented, are crude and integrated systems might be superior, at least in the initial stage of market evolution. A thorough evaluation of unbundled and integrated market systems is given in [2].

This paper analyzes the characteristics of simultaneous auctions of integrated systems and the pricing mechanisms for simultaneously procured energy and Ancillary Services (A/S) using an AC Optimal Power Flow (OPF)-based formulation. The New York ISO (NYISO) has implemented the approach of simultaneous auction by an AC OPF formulation [3]. The California ISO is also in the process of implementing a similar formulation [4][5]. However, a detailed formulation of the problem and a clear explanation of the implications of the simultaneous formulation have not been forthcoming. This paper presents a detailed mathematical formulation of the simultaneous auction of energy and A/S and a rigorous analysis of the characteristics of the prices defined by the resulting Lagrange multipliers. The theoretical analysis provided by this paper has helped us validate intuitive beliefs and insights gained over the course of many years of designing, implementing and running energy markets, and identify and discard misconceptions that unfortunately are still prevalent in the design of wholesale energy markets. For example, this paper shows that the provision of Regulation-
Traditional OPF formulations and their solution characteristics are well described in [6], [7], [8], [9] and [10]. The relationship between nodal prices and transmission shadow prices is also well analyzed in [11], [12] and [13]. The sequential auction for A/S that is currently deployed by the California ISO is described in [14] and [15]. The optimization formulations that form the basis of the simultaneous auctions at the New York ISO and New England ISO are described in [16] and [17]. Few other papers, such as [18], are presenting the methodology of pricing energy and A/S using OPF formulations. However, these papers do not provide sufficient details to allow a thorough analysis of the relationships among the prices for energy and A/S. In particular, we could not find in the literature the definition and analysis of prices for A/S when economic substitution of services is required. Although the Rational Buyer approach in [15] for procuring A/S allows economic substitution of services, it does not simultaneously optimize procurement of energy and A/S. Moreover, the rational buyer formulation minimizes total payment instead of total cost; and the A/S prices so produced have exposed the California ISO to financial neutrality problems.

The rest of the paper is organized as follows. Section II presents the AC OPF formulation used in integrated systems for the simultaneous auction of energy and A/S. Section III provides insights on the characteristics of locational marginal prices for energy and A/S. Section IV illustrates the concepts by examples and Section V concludes the paper.

II. FORMULATION FOR SIMULTANEOUS ENERGY AND ANCILLARY SERVICE AUCTION

The proposed market design is based on Locational Marginal Pricing (LMP) of energy and regional Ancillary Service Marginal Pricing (ASMP) of Regulation Up (Reg-Up), Regulation Down (Reg-Down), Spinning Reserve (Spin), and Non-Spinning Reserves (Non-Spin). The California ISO plans to operate such a market in the future. Specifically it will run a Day-Ahead (DA) market and an Hour-Ahead (HA) market to auction both energy and A/S. In both forward markets, the LMPs and the ASMPs are determined by an AC Optimal Power Flow function that is part of a Security Constrained Unit Commitment (SCUC) program. The AC OPF function optimally dispatches power and procures reserve capacity from the already committed generation, participating interchange, and dispatchable load while satisfying AC power flow equations, A/S requirements, transmission and operating constraints. Contingencies can be formulated by repeating the power flow equations and other voltage-dependent constraints for each contingency that needs to be considered. However, without loss of generality and in order to simplify the exhibition of the paper, the formulation for the normal operating condition is used in this paper. Although the Unit-Commitment (UC) problem is an integral part of the new CAISO market design, the description of the UC problem is not within the scope of this paper. The focus of this paper is the AC OPF that determines the final schedules and prices based on the UC results. Inter-temporal constraints are not modeled explicitly in the formulation. In the general case, the AC OPF function needs to be integrated with an AC OPF function to resolve inter-temporal constraint violations. Certain algorithms for modeling ramping constraints in the UC formulation can minimize the number of iterations at the expense of increased dimension. However, the focus of this paper is to define and solve the problem for pricing assuming a set of scheduled units rather than developing algorithms for solving the general scheduling problem in its entirety.

A. Objective

The objective of the AC OPF formulation is to minimize the sum of energy costs and A/S costs over a prescribed settlement interval. The settlement interval is usually one hour for the integrated forward markets. Each energy cost function is a piecewise linear convex curve. Each A/S cost function is a linear function represented by the product of the procured quantity and the bid price. The energy cost curve for each resource is either market-based bids submitted by the resources or cost-based bids calculated from heat-rate, gas price index, and operation and maintenance costs. The minimum load cost and the startup cost are not included in the OPF formulation because they have been considered in the unit commitment stage of the SCUC formulation. The objective for the AC OPF is to minimize the sum of the following components: (i) Energy Costs, (ii) Reg-Up Costs, (iii) Spin Costs, (iv) Non-Spin Costs, and (v) Reg-Down Costs. Mathematically the objective is to minimize:

$$C_{\text{Total}}(\cdot) = \sum_{i=1}^{N-1} C_i(P_i) + C_N[P_N(\mathbf{x})] + \sum_{\omega \in I_{RU}} C_{\text{RU}}(RU_i)$$

+ \sum_{\omega \in I_{SP}} C_{\text{SP}}(SP_i) + \sum_{\omega \in I_{NS}} C_{\text{NS}}(NS_i) + \sum_{\omega \in I_{RD}} C_{\text{RD}}(RD_i)$$

(1)

where the symbols are defined as follows:

- $C_{\text{Total}}(\cdot)$: Total cost of energy and A/S
- $C_i(P_i)$: Energy cost function at node $i$
- $C_{\text{NS}}(NS_i)$: Non-Spin cost function at node $i$
- $C_{\text{RD}}(RD_i)$: Reg-Down cost function at node $i$
- $C_{\text{RU}}(RU_i)$: Reg-Up cost function at node $i$
- $C_{\text{SP}}(SP_i)$: Spin cost function at node $i$
- $C_N[P_N(\mathbf{x})]$: Energy cost function of reference node
- $I_{NS}$: Set of nodes providing non-spin reserve
- $I_{RD}$: Set of nodes providing Reg-Down
- $I_{RU}$: Set of nodes providing Reg-Up
- $I_{SP}$: Set of nodes providing Spin
- $NS_i$: Non-spin reserve provided by node $i$
- $RD_i$: Reg-down capacity provided by node $i$
- $RU_i$: Reg-Up capacity provided by node $i$
- $SP_i$: Spin capacity provided by node $i$

B. Power Balance Constraint

The power balance constraints are described by the AC power flow equations. The demand is the scheduled quantity in the
forward energy market. To simplify the presentation, it is assumed in this paper that each node has at maximum one resource. In a practical implementation when there are multiple resources connected to the same bus, each resource injection is modeled as a separate variable and the bus injection is considered to be the sum of all the resource injections. Given a power system with $N$ nodes, we number the nodes as follows for convenience of reference:

- PQ nodes (i.e., load or generator operating at reactive power limit) are numbered from 1 to $N_d$.
- PV nodes (i.e., generator or load with voltage control) are numbered from $N_d+1$ to $N_d+N_g$.
- Slack node (i.e., the reference node) is numbered as the last bus, $N$.

The set of AC power flow equations generally consists of:

- $N_d$ equations that describe the active power balance at the PQ nodes.
- $N_g$ equations that describe the active power balance at the PV nodes.
- $N_d$ equations that describe the reactive power balance at the PQ nodes.

Mathematically, the equations are described as follows:

$$\Delta P_i(x, P_i) = P_i(x) - P_i = 0 \quad \text{for } i = 1, 2, ..., N-1 \quad (2)$$

$$\Delta Q_i(x, Q_i) = Q_i(x) - Q_i = 0 \quad \text{for } i = 1, 2, ..., N_d \quad (3)$$  

where $x=[\theta_1, \theta_2, ..., \theta_{N_d}, V_1, V_2, ..., V_{N_d}]^T$ representing the voltage phase angles $\theta_i$ and magnitudes $V_i$. Equation (2) represents active power balancing equations at all nodes except the reference node and $P_i$ denotes active power injection at node $i$. Equation (3) represents reactive power balancing equations at the PQ nodes and $Q_i$ denotes reactive injection at node $i$. The active power loss of the system is determined by (4)

$$\sum_{i=1}^{N} P_i(x) - P_{loss} = 0 \quad (4)$$

where $P_{loss}$ denotes the active power transmission loss of the system.

C. Capacity Reserve Constraints

Reg-Up, Reg-Down, Spin, Non-Spin are procured optimally to minimize the total cost of energy and reserves. Voltage Support and Black Start services are procured by resource specific agreements between the ISO and the suppliers, which are not part of the optimization process.

The capacity reserve constraints are inequality constraints to ensure the right amount of capacity is procured according to the prescribed A/S requirements by A/S regions that are defined off-line based on the ISO load forecast and other operating system conditions consistent with NERC standards. The resources within the same A/S region must meet a prescribed portion of the regional A/S requirements. Moreover, the following substitutions are allowed: (i) Reg-Up can meet Spin and Non-Spin requirements; (ii) Spin can meet Non-Spin requirements.

1) Regulation Up Requirement

Equation (5) specifies the amount of Reg-Up that needs to be procured from generators in each region $j$:

$$R_{j}^{RU} - \sum_{i \in I_{RU} \cap Z_j} RU_i \leq 0 \quad (5)$$

where the symbols are defined as follows:

- $R_{j}^{RU}$ Requirement of Reg-Up in region $j$
- $Z_j$ Set of nodes in region $j$

2) Spinning Reserve Requirement

Equation (6) specifies the total amount of Reg-Up and Spin that needs to be procured from resources in each region $j$:

$$R_{j}^{RU} + R_{j}^{SP} - \sum_{i \in I_{RU} \cap Z_j} RU_i - \sum_{i \in I_{SP} \cap Z_j} SP_i \leq 0 \quad (6)$$

where $R_{j}^{SP}$ denotes the requirement of Spin in region $j$

3) Non-Spinning Reserve Requirement

Equation (7) specifies the total amount of Reg-Up, Spin and Non-Spin that needs to be procured from resources in each region $j$:

$$R_{j}^{RU} + R_{j}^{SP} + R_{j}^{NS} - \sum_{i \in I_{RU} \cap Z_j} RU_i - \sum_{i \in I_{SP} \cap Z_j} SP_i - \sum_{i \in I_{NS} \cap Z_j} NS_i \leq 0 \quad (7)$$

where $R_{j}^{NS}$ denotes the requirement of Non-Spin in region $j$

4) Regulation Down Requirement

Equation (8) specifies the amount of Reg-Down, $R_{j}^{RD}$, that needs to be procured from generators in each region $j$:

$$R_{j}^{RD} - \sum_{i \in I_{RD} \cap Z_j} RD_i \leq 0 \quad (8)$$

5) Regulation Up Bid Limit

The awarded quantity for Reg-Up for each generator $i$ must be non-negative and may not be greater than an upper limit, $RU_i^{Max}$, which represents the bid limit or physical limits such as ramp rates.

$$0 \leq RU_i \leq RU_i^{Max} \quad (9)$$

6) Spinning Bid Limit

Similarly, the awarded quantity for Spin is non-negative and limited by an upper limit, $SP_i^{Max}$, as follows:

$$0 \leq SP_i \leq SP_i^{Max} \quad (10)$$

7) Non Spinning Bid Limit

The awarded quantity for Non-Spin is also non-negative and limited by an upper limit, $NS_i^{Max}$, as follows:

$$0 \leq NS_i \leq NS_i^{Max} \quad (11)$$

8) Regulation Down Bid Limit

The awarded quantity for Reg-Down is also non-negative and limited by an upper limit, $RD_i^{Max}$, as follows:

$$0 \leq RD_i \leq RD_i^{Max} \quad (12)$$

D. Supply Constraints

Active power supplies are limited by available capacity and ramping capability as follows.
1) Active Power Maximum Limit
The total power output plus the capacity reserves for Reg-Up, Spin and Non-Spin from each resource \(i\) should not exceed its maximum operating limit, i.e.,
\[
P_i + RU_i + SP_i + NS_i - P_i^{\text{Max}} \leq 0
\]
where \(P_i^{\text{Max}}\) is the maximum operating limit of the resource at node \(i\) for the particular hour.

2) Active Power Minimum Limit
A generator once committed must maintain a minimum output. In addition, if a generator provides Reg-Down, it must produce additional power to make room for Reg-Down capacity. Such constraints are described as follows:
\[
P_i^{\text{Min}} - P_i - RD_i \leq 0
\]
where \(P_i^{\text{Min}}\) is the minimum load of the resource at node \(i\).

3) Ten-minute Ramp Limit
The total amount of provision for Reg-Up, Spin and Non-Spin from a resource, \(i\), is limited by its ramping capability within 10 minutes.
\[
RU_i + SP_i + NS_i - 10 \leq 0
\]
where \(RU_i^{\text{Min}}\) and \(RU_i^{\text{Min}}\) are the ramp rates of resource \(i\) in MW/minute for providing Reg-Up and operating reserves.

E. Network Constraints
Network constraints in this paper include the following types:
- Reactive power supply limits
- Voltage magnitude and phase angle limits
- Branch flow limits, and
- Other network limits such as nomograms [19]

Any network constraint \(k\) except tie-line constraints can be represented in the following form:
\[
F_k(x) - F_k^{\text{Max}} \leq 0
\]
where \(F_k(x)\) is the quantity that is limited by constraints \(k\); and \(F_k^{\text{Max}}\) is the upper limit of the quantity described by constraint \(k\). Special examples of (16) include reactive power supply limits and voltage limits as follows:
\[
Q_i^{\text{Min}} - Q_i^{\text{Max}} \leq 0 \quad \text{for } i = N_d+1, N_d+2, \ldots, N_d+N_g
\]
\[
Q_i^{\text{Min}} - Q_i^{\text{Max}} \leq 0 \quad \text{for } i = N_d+1, N_d+2, \ldots, N_d+N_g
\]
where \(Q_i^{\text{Max}}\) is the upper limit of reactive power injection at node \(i\); and \(Q_i^{\text{Min}}\) is the lower limit of the reactive power injection at node \(i\). Note \(Q_i(x)\) at PV nodes are functions of voltage variables.
\[
V_i - V_i^{\text{Max}} \leq 0 \quad \text{for } i = 1, 2, \ldots, N_d
\]
\[
V_i^{\text{Min}} - V_i \leq 0 \quad \text{for } i = 1, 2, \ldots, N_d
\]
where \(V_i^{\text{Max}}\) is the upper limit of voltage magnitude at node \(i\); and \(V_i^{\text{Min}}\) is the lower limit of the voltage magnitude at node \(i\).
\[
\theta_i - \theta_i^{\text{Max}} \leq 0 \quad \text{for } i = 1, 2, \ldots, N-1
\]
\[
\theta_i^{\text{Min}} - \theta_i \leq 0 \quad \text{for } i = 1, 2, \ldots, N-1
\]
where \(\theta_i^{\text{Max}}\) is the upper limit of voltage phase angle at node \(i\); and \(\theta_i^{\text{Min}}\) is the lower limit of the voltage phase angle at node \(i\).

Equation (16) also describes transmission limits in the following forms:
- Branch or Branch Group Limit: Power flow limit on an individual transmission branch or a branch group represented by a constant.
- Nomogram: Power flow limit on a transmission interface represented by a function of other variables such as output of a certain group of generators, load of a certain area, or flows on other transmission interfaces. Such functions are modeled by piece-wise linear functions.

Reg-Up, Spin and Non-Spin provided from resources outside of the ISO control area can compete with energy schedules for transmission usage on tie lines based on their capacity bids. When A/S compete with energy schedules for the use of tie-line \(k\), the constraint is described as follows:
\[
F_k(x) + \sum_{i \in T_k} (RU_i + SP_i + NS_i) - F_k^{\text{Max}} \leq 0
\]
where \(RU_i\), \(SP_i\) and \(NS_i\) are quantities of Reg-Up, Spin and Non-Spin capacity from resource \(i\) provided across tie line \(k\); \(T_k\) denotes the set of resources that compete for the use of tie-line \(k\).

III. Definitions, Characteristics and Components of Energy and Ancillary Service Prices

A. Lagrange Function
The Lagrange function in our formulation is as follows.
\[
L = \sum_{i=1}^{N-1} C_i(P_i) + C_N[P_N(x)] + \sum_{i \in R_U} C_i^{RU}(RU_i) + \sum_{i \in S_P} C_i^{SP}(SP_i) + \sum_{i \in N_S} C_i^{NS}(NS_i) + \sum_{i \in R_D} C_i^{RD}(RD_i) + \sum_{i=1}^{N-1} \lambda_i [P_i(x) - P_i] + \sum_{i=1}^{N_d} \lambda_i [Q_i(x) - Q_i] + \sum_{i \in R_U \cap C_j} R_j^{RU} - \sum_{i \in S_P \cap C_j} R_j^{SP} + \sum_{i \in N_S \cap C_j} R_j^{NS} + \sum_{i \in R_D \cap C_j} R_j^{RD}
\]

(Reg-Up Cost) (Spin Cost) (Non-Spin Cost) (Reg-Down Cost) (Active Power Balance) (Reactive power balance at PQ nodes) (Reg-Up Requirement) (Spin Requirement)
Suppose the power balance equation at node \( i \) equals to the incremental cost for supplying an additional MW of power at node \( i \). Suppose the power balance equation at node \( i \) is perturbed by \( \Delta P_i = P_i(x) - P_i \). The incremental cost for the perturbation at the optimal point is as follows:

\[
\frac{\partial L}{\partial P_i} = \frac{\partial L}{\partial [P_i(x) - P_i]} = \lambda_i
\]  

where \( \lambda_i \) is the LMP for active power at node \( i \). As is shown in the literature [3][4], each nodal price can be decomposed into three components: (i) incremental cost at the reference bus, (ii) incremental cost of thermal transmission losses, and (iii) incremental cost of network constraints which include transmission constraints, power supply constraints, voltage constraints and phase angle constraints, i.e.,

\[
\lambda_i = \lambda_N - \lambda_{SP} - \sum_k \mu_k S_{ki}
\]  

where:

\[
\lambda_N = \frac{\partial C_N}{\partial P_i}
\]  

System marginal cost of energy at the reference node

\[
L_i = \frac{\partial P_{loss}}{\partial P_i}
\]  

The \( i \)-th Loss Contribution Factor.

\[
S_{ki} = \frac{\partial F_k}{\partial P_i}
\]  

The sensitivity of the quantity limited by constraint \( k \) with respect to active power injected into node \( i \) and withdrawn at the reference node.

The LMPs are not affected by the choice of the reference node because the losses are optimally distributed according to the supply bids by the OPF. However, the Loss Contribution Factors are affected by the choice of the reference node. To avoid the controversy regarding the selection of the reference node, we recommend using the entire LMP value at each node rather than its individual components for settlement purposes. If the loss component of the LMP value at each bus has to be settled separately from energy for commercial, regulatory, or other reasons, a Load Center Penalty Factor [20] approach can be used.

### C. Ancillary Service Marginal Prices (ASMP)

#### 1) Definitions

The ASMP for an A/S in region \( j \) is the incremental cost for meeting an additional MW of the requirement for the A/S in region \( j \) as follows:

\[
\frac{\partial L}{\partial R_{RU_j}} = \lambda_{RU_j} + \lambda_{SP_j} + \lambda_{NS_j}
\]  

(Reg-Up Price)  

\[
\frac{\partial L}{\partial R_{SP_j}} = \lambda_{SP_j} + \lambda_{NS_j}
\]  

(Spin Price)  

\[
\frac{\partial L}{\partial R_{NS_j}} = \lambda_{NS_j}
\]  

(Non-Spin Price)  

\[
\frac{\partial L}{\partial R_{RD_j}} = \lambda_{RD_j}
\]  

(Reg-Down Price)

#### 2) Properties for Neutrality

It is shown next that when a higher quality service is procured to meet the requirement of a lower quality service, e.g., Reg-Up is procured to meet the Spin requirement, the ASMPs for the two services are equal. Suppose Reg-Up is used to meet the Spin requirement in region \( j \); that is:

\[
\sum_{j} \lambda_{NS_j} = \sum_{j} \lambda_{SP_j} + \sum_{j} \lambda_{RD_j}
\]
According to the Kuhn-Tucker conditions, the following holds true:

$$\lambda_{j}^{RU} \left( R_{j}^{RU} - \sum_{i \in I_{RU} \cap Z_{j}} RU_{i} \right) = 0 \Rightarrow \lambda_{j}^{RU} = 0$$  \hspace{1cm} (31)$$

Consequently, the ASMPs for Reg-Up and Spin in region j are the same, i.e.,

$$\frac{\partial L}{\partial R_{j}^{RU}} = \frac{\partial L}{\partial R_{j}^{SP}} = \lambda_{j}^{SP} + \lambda_{j}^{NS}$$  \hspace{1cm} (32)$$

Similar analysis can be carried out for substitution among other A/S.

This important property allows A/S costs incurred to the ISO be allocated to market participants without the neutrality problem that occurs to the rational buyer approach [15]. To illustrate this point, let’s continue with the above scenario but without the neutrality condition, the ASMPs for Reg-Up and Spin in region j are the same, i.e.,

$$\frac{\partial L}{\partial R_{j}^{RU}} = \frac{\partial L}{\partial R_{j}^{SP}} = \lambda_{j}^{SP} + \lambda_{j}^{NS}$$  \hspace{1cm} (33)$$

The total charge to the market participants is:

$$Charge = \frac{\partial L}{\partial R_{j}^{RU}} R_{j}^{RU} + \frac{\partial L}{\partial R_{j}^{SP}} R_{j}^{SP}$$  \hspace{1cm} (34)$$

When Reg-Up is used to meet the requirement for Spin only, we have:

$$\sum_{i \in I_{RU} \cap Z_{j}} RU_{i} > R_{j}^{RU}, \quad \sum_{i \in I_{SP} \cap Z_{j}} SP_{i} < R_{j}^{SP}$$  \hspace{1cm} (35)$$

The neutrality imbalance for the ISO is the difference between the payment and the charge as shown in (37); it is zero if and only if the prices for Reg-Up and Spin are equal.

$$Payment - Charge = \left( \frac{\partial L}{\partial R_{j}^{RU}} - \frac{\partial L}{\partial R_{j}^{SP}} \right) \sum_{i \in I_{RU} \cap Z_{j}} RU_{i} - R_{j}^{RU}$$  \hspace{1cm} (36)$$

**D. Opportunity Costs**

1) **Opportunity Costs for Provision of Regulation Up, Spinning Reserve and Non-Spinning Reserve**

Reg-Up, Spin and Non-Spin are referred to as upward reserves. A supplier of upward reserves may sell less energy in a forward market (i.e., DA or HA) than it would have been economic for it to sell because of the provision of upward reserves. This happens if and only if the resource is constrained by its maximum capacity. Suppose resource i participates in the energy and only in the Non-Spin markets. According to the Kuhn-Tucker condition,

$$\frac{\partial L}{\partial NS_{i}} = \frac{\partial C_{i}^{NS}}{\partial NS_{i}} - \lambda_{i}^{NS} + \pi_{i}^{Max} + \alpha_{i}^{NS} - \beta_{i}^{NS} + \alpha_{i}^{OP} + \mu_{i} = 0$$  \hspace{1cm} (37)$$

$$\frac{\partial L}{\partial P_{i}^{l}} = \frac{\partial C_{i}}{\partial P_{i}^{l}} - \lambda_{i}^{l} + \pi_{i}^{Max} - \pi_{i}^{Min} = 0$$  \hspace{1cm} (38)$$

In order to demonstrate the opportunity costs, let’s assume, without loss of generality, that resource i is an internal resource (i.e., eliminating $\mu_{i}$ from (38)), providing a positive amount of Non-Spin (i.e., $\beta_{i}^{NS} = 0$), not constrained by its bid quantity (i.e., $\alpha_{i}^{NS} = 0$), not constrained by ramp rate (i.e., $\alpha_{i}^{OP} = 0$), and not constrained by its minimum load limit (i.e., $\pi_{i}^{Min} = 0$). Under these assumptions,

$$\lambda_{j}^{NS} = \frac{\partial C_{i}^{NS}}{\partial NS_{i}}$$  \hspace{1cm} (39)$$

$$\pi_{i}^{Max} = \lambda_{i}^{l} - \frac{\partial C_{i}}{\partial P_{i}^{l}}$$  \hspace{1cm} (40)$$

Now if the resource is not limited by its maximum capacity (i.e., $\pi_{i}^{Max} = 0$), the ASMP for Non-Spin in region j and the LMP for node i are determined to be the marginal cost of Non-Spin and the marginal cost of energy independently. However, if the marginal resource i for Non-Spin is limited by its maximum capacity (i.e., $\pi_{i}^{Max} > 0$), we have,

$$\lambda_{j}^{NS} = \frac{\partial C_{i}^{NS}}{\partial NS_{i}} + \left( \lambda_{i}^{l} - \frac{\partial C_{i}}{\partial P_{i}^{l}} \right)$$  \hspace{1cm} (41)$$

Equation (42) shows that the ASMP for Non-Spin consist of two components: (i) the marginal Non-Spin bid price, and (ii) a component that represents the opportunity cost to resource i for the provision of Non-Spin instead of energy. This analysis can be done for other upward A/S or under more complex conditions. The conclusion, however, remains the same.

2) **Opportunity Costs for Provision of Regulation Down**

A supplier of Reg-Down may have to sell more energy in a forward market (i.e., DA or HA) than it would have been economic for it to sell because of the provision for Reg-Down. This happens if and only if the resource is constrained by its minimum load limit. Suppose resource i participates in the Reg-Down market only. According to the Kuhn-Tucker condition,

$$\frac{\partial L}{\partial RD_{i}} = \frac{\partial C_{i}^{RD}}{\partial RD_{i}} - \lambda_{i}^{RD} + \pi_{i}^{Min} + \alpha_{i}^{RD} - \beta_{i}^{RD} = 0$$  \hspace{1cm} (42)$$

In order to derive the opportunity costs, assume that resource i is providing a positive amount of Reg-Down (i.e., $\beta_{i}^{RD} = 0$), not constrained by its upper limit (i.e., $\alpha_{i}^{RD} = 0$), and not constrained by its maximum capacity (i.e., $\pi_{i}^{Max} = 0$). Under these assumptions,

$$\lambda_{j}^{RD} = \frac{\partial C_{i}^{RD}}{\partial RD_{i}} + \pi_{i}^{Min}$$  \hspace{1cm} (43)$$
\[ \lambda^\text{Min}_i = \frac{\partial C_i}{\partial P_i} - \lambda_i \]  

(45)

Now if the resource is not limited by its minimum capacity (i.e., \( \pi^\text{Min} = 0 \)), the ASMP for Reg-Down in region \( j \) and the LMP for node \( i \) are determined to be the marginal cost of Reg-Down and marginal cost of energy independently. However, if the resource is limited by its minimum capacity (i.e., \( \lambda^\text{Min}_i > 0 \)), we have,

\[ \lambda^\text{RD}_j = \frac{\partial C^\text{RD}_j}{\partial \lambda^\text{RD}_j} + \left( \frac{\partial C_i}{\partial P_i} - \lambda_i \right) \]

(46)

Equation (46) shows that the ASMP for Reg-Down consists of two components: (i) the marginal Reg-Down bid price, and (ii) a component that represents the opportunity cost to resource \( i \) due to selling energy below its bid price as a consequence of providing Reg-Down.

**E. Price for Transmission**

1) **Definition of Shadow Price**

The shadow price for transmission constraint \( k \) is determined to be the marginal cost of constraint \( k \) as follows:

\[ \frac{\partial L}{\partial F^\text{Max}_k} = \mu_k \]  

(47)

2) **Price for Point-To-Point Transmission**

The price for using the transmission system to deliver one MW from node \( i \) to node \( j \) is defined as follows:

\[ \tilde{\lambda}_j - \lambda_j = \lambda_N (L_j - L_i) + \sum_k \mu_k (S_{ki} + S_{kj}) \]  

(48)

The first term on the right hand side of (48) represents the cost of losses attributable to the transaction between node \( i \) and node \( j \); the second term represents the cost of transmission constraints including thermal limits on transmission branches (groups), reactive power limits, voltage limits and other general nomogram constraints.

3) **Price for Network Service Transmission**

To avoid double subscripts in notations, any network service right can be described as the right of sending (\( p_1, p_2, \ldots, p_i \)) % of one MW at nodes (1, 2, \ldots, \( s \)) and receiving (\( p_{s+1}, p_{s+2}, \ldots, p_{s+r} \)) % of one MW at nodes (\( s+1, s+2, \ldots, s+r \)). Using this notation, the price for paying any network service right is described as follows:

\[ \sum_{j=s+1}^{s+r} \lambda_j p_j - \sum_{i=1}^{s} \lambda_i p_i = \lambda_N \left( \sum_{i=1}^{s} L_i p_i - \sum_{j=s+1}^{s+r} L_j p_j \right) + \sum_k \mu_k \left( \sum_{i=1}^{s} S_{ki} p_i - \sum_{j=s+1}^{s+r} S_{kj} p_j \right) \]  

(49)

The first term on the right hand side of (49) represents the cost of losses attributable to the transactions associated with the transmission service; the second term represents the cost of transmission constraints including thermal limits on transmission branches (groups), reactive power limits, voltage limits and general nomogram constraints.

**4) Total Congestion Revenue from Energy Settlement**

The total congestion revenue collected by the ISO through LMP settlements for energy has the following relationship:

\[ CR = \sum_{i=1}^{N} (\tilde{\lambda}_i P_i) = \sum_k \mu_k F_k^\text{Max} + \lambda_N \left( \sum_{i=1}^{N} L_i p_i - P_{\text{loss}} \right) \]  

(50)

The left hand side of (50) indicates that the total amount of congestion revenue is the leftover from the energy settlement at all the nodes. Replacing the \( \bar{\lambda}_i \) in (50) by the expression in (25), one can obtain, after some manipulation, the right hand side of (50). On the right hand side of (50), the first term represents the revenue associated with transmission congestion and resource limit violations. The second term represents an over collection from the compensation of marginal losses.

5) **Transmission Price for Importing Ancillary Services**

Since A/S requirements are determined according to load and export quantities of the internal control area, A/S provided from outside of the control area participate in the A/S auction in the A/S regions within the control area. A/S imports are settled by the same regional ASMPs that are used by the internal resources. However, importing Reg-Up, Spin and Non-Spin through congested tie lines incurs congestion charges to the importers. This charge is priced by the shadow price on the tie line.

Suppose an external resource \( i \) is providing Non-Spin over tie line \( k \). Consider the same Kuhn Tucker condition shown in (38). Since imports are not limited by capacity or ramp rates, \( \alpha^\text{OP}_i = \pi^\text{Max} = \pi^\text{Min} = 0 \) and since the import of Non-Spin is positive, \( \beta^\text{NS}_i = 0 \). The only constraints that are potentially active are the Non-Spin max bid limit and the tie-line flow limit. Therefore,

\[ \lambda^\text{NS}_j = \frac{\partial C^\text{NS}_i}{\partial NS_j} + \alpha^\text{NS}_i + \mu_k \]  

(51)

After applying the congestion charge to resource \( i \), the actual price received by resource \( i \) for providing Non-Spin is as follows:

\[ \lambda^\text{NS}_j - \mu_k = \frac{\partial C^\text{NS}_i}{\partial NS_j} + \alpha^\text{NS}_i \]  

(52)

On the right hand side of (52), the first term represents the bid price of resource \( i \) for Non-Spin, and the second term represents the supplier’s surplus, which is zero if resource \( i \) is the marginal supplier for Non-Spin in region \( j \).

**IV. Examples**

The examples are designed to facilitate understanding of the paper rather than presenting simulation results. In order to focus on the key issues, a simple 3-node DC network as shown in Fig. 1 is used. The network has 3 identical branches; each branch is rated at 50 MW in both directions. The load \( L_3 \) has a fixed schedule of 150 MW. All the 3 generators \( G_1, G_2 \) and \( G_3 \) can operate between 0 and 100 MW with infinite ramping limits and general nomogram constraints.
capability. It is further assumed that all the generators are located within the same A/S region inside the control area; and therefore the Spin reserves do not compete with energy for the congested transmission network. The Spin requirement, $R^{sp}$, is 10 MW in Case 1 and 30 MW in Case 2, which are used to illustrate ASMPs without and with opportunity costs.

**Case 1: $R^{sp} = 10$ MW**

As is shown in Table I, all generators offer both energy and Spin reserve; $G_1$ is the most economic energy and Spin provider. Since the three branches are identical, to supply each MW of power from $G_1$ to $L_3$, 2/3 MW goes through Branch #1–#3; and 1/3 MW goes through Branches #1–#2 and #2–#3. The following simplified DC OPF problem is formulated:

Minimize:

$$10P_1 + 30P_2 + 45P_3 + 5SP_1 + 15SP_2 + 40SP_3 \quad (53)$$

Subject to the constraints:

- **Power Balance**: $P_1 + P_2 + P_3 = 150$ \quad (54)
- **Spin Required**: $SP_1 + SP_2 + SP_3 \geq R^{sp}$ \quad (55)
- **Flow (#1→#2)**: $-50 \leq (1/3) P_1 - (1/3) P_2 \leq 50$ \quad (56)
- **Flow (#1→#3)**: $-50 \leq (2/3) P_1 + (1/3) P_2 \leq 50$ \quad (57)
- **Flow (#2→#3)**: $-50 \leq (1/3) P_1 + (2/3) P_2 \leq 50$ \quad (58)
- **Capacity limits**: $0 \leq P_i, SP_i \leq 100$ for $i = 1, 2, 3$ \quad (59)
- **Lower Bounds**: $0 \leq P_i$ and $0 \leq SP_i$ for $i = 1, 2, 3$ \quad (60)

**Table I**

<table>
<thead>
<tr>
<th>Resources</th>
<th>Energy Bid</th>
<th>Energy Bid Price ($/MW)</th>
<th>Spin Bid</th>
<th>Spin Bid Price ($/MWh)</th>
<th>Total Capacity (MW)</th>
<th>Energy Schedule (MW)</th>
<th>LMP ($/MW)</th>
<th>Spin Award ($/MWh)</th>
<th>Opportunity Cost ($/MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>10</td>
<td>5</td>
<td>100</td>
<td>75</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G_2$</td>
<td>30</td>
<td>15</td>
<td>100</td>
<td>0</td>
<td>27.5</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$G_3$</td>
<td>45</td>
<td>40</td>
<td>100</td>
<td>75</td>
<td>45</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$L_3$</td>
<td>Fixed</td>
<td>N/A</td>
<td>150</td>
<td>150</td>
<td>45</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The solution in this case is obtained by solving a simple LP problem; the results are given in Table I. Since $G_1$ and $G_2$ compete for the use of Branch #1→#3 and $G_1$ is far more competitive than $G_2$ in terms of providing energy, $G_1$ is awarded a 75 MWh energy schedule to fully utilize the 50 MW capacity of Branch #1→#3; $G_1$ picks up the other 75 MW of load. The resulting LMPs at Node #1 and Node #3 are set by $G_1$ and $G_3$ at $10$/MWh and $45$/MWh, respectively.

**Case 2: $R^{sp} = 30$ MW**

The solution in this case is also obtained by solving the LP problem; the results are given in Table II. Since $G_1$ has reached its full capacity by providing 70 MW of energy and 30 MW of Spin, it is not a marginal unit and cannot set the marginal price. The resulting LMPs at Node #2 and Node #3 are set by $G_2$ and $G_3$ at $30$/MWh and $45$/MWh, respectively. The LMP for Node #1 is calculated using (25) disregarding the loss component, i.e., $\lambda_1 = \lambda_1 - (1/3)\mu_{l\rightarrow3}$ where $\mu_{l\rightarrow3}$ is obtained from $\lambda_2 = \lambda_2 - (2/3)\mu_{l\rightarrow3}$. Since $\lambda_2 = $30$/MWh and $\lambda_3 = $45$/MWh, $\mu_{l\rightarrow3} = $45*(2/3)*45 = $15$/MWh. The ASMP for Spin is set by $G_1$ at $5$/MWh. There is no opportunity cost for $G_1$ in this case because $G_1$ still has unused capacity after providing energy and the Spin.

**Table II**

<table>
<thead>
<tr>
<th>Resources</th>
<th>Energy Bid</th>
<th>Energy Bid Price ($/MW)</th>
<th>Spin Bid</th>
<th>Spin Bid Price ($/MWh)</th>
<th>Total Capacity (MW)</th>
<th>Energy Schedule (MW)</th>
<th>LMP ($/MW)</th>
<th>Spin Award ($/MWh)</th>
<th>Opportunity Cost ($/MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>10</td>
<td>5</td>
<td>100</td>
<td>75</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G_2$</td>
<td>30</td>
<td>15</td>
<td>100</td>
<td>0</td>
<td>27.5</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$G_3$</td>
<td>45</td>
<td>40</td>
<td>100</td>
<td>75</td>
<td>45</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$L_3$</td>
<td>Fixed</td>
<td>N/A</td>
<td>150</td>
<td>150</td>
<td>45</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**V. Conclusions**

A detailed formulation of simultaneous energy and A/S auctions for integrated market systems is presented. Rigorous definitions are given for the Locational Marginal Prices (LMP) for energy and the Ancillary Service Marginal Prices (ASMP) when economic substitution among A/S is required. The paper analyzes and provides insights on the properties of the prices and the relationships among the prices that are determined by this optimal power flow formulation. The following findings resulted from the analysis of the prices resulting from our formulation are not at all intuitive, and in some cases are counter-intuitive:

- Provision of Reg-Down incurs opportunity cost to the provider if the unit is constrained by a minimum schedule limit in order to make room for the provision of Reg-Down and therefore has to provide energy at a price below its bid.
- Provision of upward A/S (i.e., Reg-Up, Spin and Non-Spin) incurs opportunity cost if the unit is operating against its maximum operating limit and therefore has to provide less energy than it is economic for it to provide in
order to leave room for the provision of the upward A/S.

- The ASMPs include compensation for opportunity costs if there is any; no additional payment is necessary for opportunity costs incurred to A/S providers.
- The ASMP for Reg-Up is never less than the ASMP for Spin. The ASMP for Spin is never less than the ASMP for Non-Spin.
- When one type of A/S capacity is procured to meet the requirement of another type of A/S capacity, the ASMPs for the two types of A/S are equal.
- The congestion charge to the A/S import across a congested interface is priced by the shadow price of the interface, which is determined by the energy bids alone.
- The total congestion revenue collected by the ISO through LMP for energy based on an AC OPF includes not only congestion charges but also an over collection of compensation for losses.

VI. ACKNOWLEDGMENT

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VII. REFERENCES


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VIII. BIOGRAPHIES

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